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The impact of income distribution on structural transformation: The role of extensive margin

Yue Liu

Central University of Finance and Economics, School of International Trade and Economics, South Collage Road 39, Beijing 100081 China

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ABSTRACT

I study the impact of income distribution on structural transformation. Empirical results suggest that income inequality induces lower share of employment in services sector, and this negative effect gets stronger as income level rises. To explain these facts, I present a multi-sector model with non-homothetic preference and heterogenous agents in terms of different income levels. In equilibrium, the individuals will not consume all the goods available in the market. While the income elasticity falls as income increases at the individual level, it may not at the aggregate level. The extensive margin of consumers is important to understand this result. Within this framework, I show that income inequality may have negative effects on an industry with income elasticity larger than 1. More importantly, this effect is getting stronger as income levels increase.

1. Introduction

Structural transformation is a stylized fact along economic growth. As GDP per capita increases, the employment share falls in agriculture, increases in services sector, while displays a hump shape in manufacturing sector (see Kuznets (1966)). Recently, the literature of structural transformation has identified several driving forces of structural change in both the demand and supply side. In the demand side, if the income elasticities differ across industries, then changes of income will induce structural transformation (e.g., Kongsamut et al., 2001; Foellmi and Zweimuller, 2008).[1]

In this paper, I discuss the impact of income distribution on structural transformation through demand side. To empirically motivate the impact of income inequality on structural transformation, I regress the employment share of services sector on the measures of income inequality and other variables. Using panel data of 17 countries over about 50 years, I find that more unequal distribution imply lower level of employment share in services sector. In addition, the negative effect gets stronger as income level increases.

These results are inconsistent with the models emphasizing the demandside mechanisms. For the models with non-homothetic preferences but linear expenditure functions, say, Kongsamut et al. (2001) and Herrendorf et al. (2013b), there should be no correlation between those two variables. For the models with very general preferences, say, the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980), the demand is a convex function of total expenditure for the goods with income elasticity larger than 1 (or the luxury goods). Therefore, these preferences should always predict positive relationships between income inequality and the share of services, as the income elasticity for services sector is generally larger than 1.

To reconcile the empirical results, I then present a model with non-homothetic preference and heterogenous agents in terms of different income levels. In equilibrium, consumers endogenously determine the sets of goods to consume. The feature of this preference is that the income elasticity of a good is initially very high, and eventually falls to below unity as income increases. This is well in line with the non-linear Engel curves.[2] However, the income elasticity for a product at the aggregate level may not decrease as income increases. This is the case for the goods consumed by only some people. Higher average income will induce more people to buy that product (the extensive margin) and existing consumers to consume more (the intensive margin). According to the preference, the income elasticity for the new consumers is very high, and this will offset the decreasing income elasticities for the old consumers. When income follows Pareto distribution, the model can be analytically solved.[3] Within this framework, I show that income inequality may have negative effects on the industry with income

E-mail address: liuy9.08@sem.tsinghua.edu.cn.

[1] Examples in the supply side are Ngai and Pissarides (2007) on productivity differences, Acemoglu and Guerrieri (2008) and Liu (2012) on differences in factor intensity across sectors. Herrendorf et al. (2013a) provides an excellent survey on this literature.

[2] The Engel curves can be non-linear in many cases (see e.g., Lewbel., 2006). Thus it is reasonable to have a model with preferences of non-linear Engel curves.

[3] Pareto distribution is a good proximate for income or wealth distribution, see Jones (2015).

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elasticity larger than 1. More importantly, this effect gets stronger as income levels increase, which is consistent with what the empirical evidences.

The intuition is as follows. With higher inequality, some people could not afford a product any more, and these people have very high income elasticities, which will greatly hurt the total demand. In addition, the density function of Pareto distribution is decreasing in income, so the number of individuals is larger for the group with relatively low income levels. Therefore, with more unequal distribution, the decrease of demand by the poorer consumers is larger than the increase of demand by the richer consumers, even for an industry with income elasticity larger than 1 at the aggregate level,

To this end, one contribution of this paper is to highlight that understanding how the macro-level variables are aggregated over individuals is important. Even if we know the income elasticity for a product is larger than unity from aggregate data, we may not immediately infer whether more unequal distribution would increase the total demand for that product or not.

In terms of the model itself, I introduce Pareto distribution into the theoretical analysis of income distribution. The functional form of Pareto distribution makes the model quite tractable and delivers closed-form solution. This is a nice property in the model with both non-homogeneous preference and heterogeneous agents, as well as multi-industries.

This paper is closely related to and follows Foellmi and Zweimuller (2008). Unlike that paper, the purpose of this paper is not to present a unified model with both structural change and balanced economic growth.[4] Instead, I focus on a static model, and discuss the impact of income inequality on structural transformation. Murphy et al. (1989), Matsuyama (2002) and Foellmi and Zweimuller (2006) use similar preferences, and discuss the impact of income distribution. Based on their insights, I characterize income distribution in a more realistic way, that is, in the form of Pareto, and show the extensive margin matters for the results.

The rest of this paper is organized as follows. Section 2 will show the empirical evidences. I present a model to provide the intuition why inequality may have negative effect on the share of services in Section 3. Section 4 will conclude this paper.

2. Empirical evidences

In this section, I present the empirical evidences about the impact of income inequality on the employment share of services sector. I employ a unbalanced panel of 17 countries from 1956 to 2004. The reason to focus on services sector rather than manufacturing is that the employment share increases monotonically increases in services, while experiences a hump-shape path, as average income increases (Herrendorf et al., 2013a). Therefore, it will deliver clearer results to work with services sector.

2.1. Regression Specification and data sources

The main objective of this paper is to check whether income inequality has significant effect on services share, and whether the effects are different at different stages of growth. So I add both the measure of income inequality and the interactions of income inequality and GDP per capita in the regressions. In addition, I follow the theoretical literature on structural transformation to choose other explanatory variables. For the demand side, the income elasticity of services goods is presumably higher than agriculture and manufacturing goods, as in Kongsamut et al. (2001). That is, as income increases, the employment or output share of services sector will increase. Therefore, GDP per capita is included as an explanatory variable. For the supply side, productivity differences across sectors will induce labor reallocations, according to Ngai and Pissarides (2007). To control the supply-side forces, I add labor productivity in agriculture, manufacturing and services sector into the regression equation.

The basic fixed-effect regression model is

$$\begin{aligned} Sershare_{it} = \; & \beta_0 + \beta_1 GDP_{it} + \beta_2 Inequality_{it} + \beta_3 GDP_{it} * Inequality_{it} \\ & + \beta_4 X_{it} + Country_i + Year_t + u_{it} \end{aligned}$$

where the subscripts i and t mean country and year, respectively. $Serviceshare_{it}$ is the employment share in services sector.[5] GDP_{it} represents GDP per capita, and $Inequality_{it}$ is the measure of income inequality. $GDP_{it} * Inequality_{it}$ is the interaction of GDP_{it} and $Inequality_{it}$. X_{it} is a vector of other control variables, including labor productivity in agriculture, manufacturing, and services sector, denoted as $prodagri$, $prodmanu$, and$prodser$. $Country_i$ and $Year_t$ are country and year fixed effects, respectively. Error term is denoted by u_{it}.

The data on employment share and sector-level labor productivities are from Duarte and Restuccia (2010). They constructed a panel dataset on PPP-adjusted real output per hour and sector-level output and hours worked for agriculture, industry, and services. The panel data include 29 countries with annual data covering the period from 1956 to 2004.[6] The reason to choose this dataset is that it provides employment shares in terms of hours worked at the sectoral level, which I believe is a better measure than that with only number of workers. In addition, the sector-level variables are adjusted to keep consistency so that they are suitable for cross-country analysis.

To better match this panel, I take advantage of the World Top Income Database (WTID) to get information on income inequality.[7] In the regressions, I choose the Inverted Pareto-Lorenz coefficient as the measure of income inequality. Larger value of this coefficient means more unequal distribution. The negative correlation between employment share of services and inequality is shown in Fig. 1.

Combining the two databsets, I get an unbalanced panel data of 17 countries from 1956 to 2004.[8] The statistics of the variables are listed in Table 1.

2.2. Results

The regression results are presented in Table 2. There are country and year fixed effects in all regressions. And the standard errors are clustered at the country level. Column (1) of Table 1 only includes GDP_{it} and $Inequality_{it}$ as the independent variables. The coefficient of $Inequality_{it}$ is negative, although not significant. However, when controlling the sector-level productivities, the effect of inequality becomes significantly negative, as in column (2). This suggests that more unequal distribution would induce lower services share. Then I add the interaction term in the regression as in Column (3). The

[4] As Herrendorf et al. (2013a) suggests that, "while the search for specifications that can simultaneously yield structural transformation and balanced growth have proven to useful in organizing research, exact balanced growth should not be imposed as a requirement moving forward".

[5] The reason to use employment share rather than output share, is that employment is relatively easier to measure, thus more accurate. Since price level at the sectoral level are different across countries, it's difficult to get data on real output at the sectoral level that are comparable both across countries and over time. In addition, the focus on employment share is consistent with most papers on structural transformation.

[6] Please refer to Duarte and Restuccia (2010) for the details about the dataset.

[7] For the construction and limitations of this database, please refer to Atkinson et al. (2011) and its webpage at http://topincomes.g-mond.parisschoolofeconomics.eu. Actually, there are not many datasets on income distribution available, especially for multi-countries over years. The World Bank Indicator contains information on Gini coefficient and income shares for different quantiles. Unfortunately, the data are just for selective years as well as selective countries, which is not feasible for panel-data analysis. Then I turn to the WTID dataset, which contains information on income distribution of the richest group, say, the groups of the top 1 percent or top 10 percent.The results are robust to other measures of income inequality, which are available upon request.

[8] The countries in this subsection are Argentina, Australia, Canada, Colombia, Denmark, France, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, U.S., and U.K.

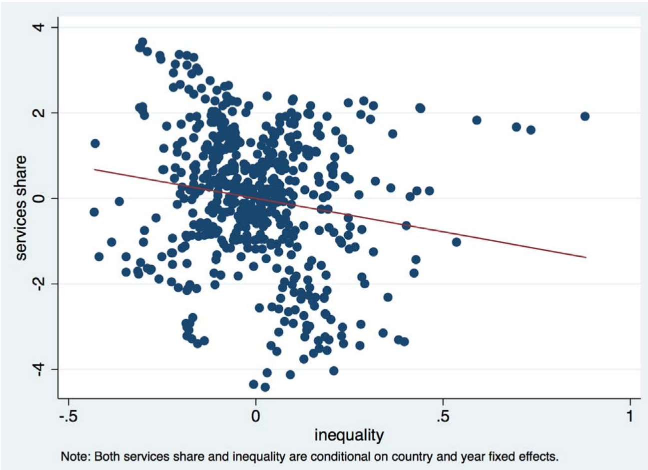

Fig. 1. The correlation between services share and inequality.

Table 1
Summary statistics.

Variable	Obs	Mean	Std. dev.	Min	Max
Ser share	579	57.8	9.01	36.7	73.3
Prod agri	579	9.401	5.530	1.922	31.05
Prod manu	579	22.95	11.79	4.241	86.48
Prod ser	579	25.77	6.989	6.912	43.26
Prod total	579	22.93	7.981	4.630	48.69
inequality	579	1.736	0.294	1.224	3.078
gdp	579	14.69	5.030	3.986	30.20

Table 2
Regression results.

VARIABLES	(1) OLS	(2) OLS	(3) OLS	(4) IV	(5) IV	(6) IV
Inequality	-1.565	-3.326**	2.668	-4.231***	2.751**	5.430***
	(2.253)	(1.147)	(2.124)	(0.457)	(1.386)	(1.822)
Interaction			-0.295**		-0.320***	-0.428***
			(0.103)		(0.0588)	(0.0740)
gdp		-0.362	0.247	-0.429***	0.253*	0.505***
		(0.222)	(0.314)	(0.0583)	(0.137)	(0.160)
Prod_agri		0.0354	0.0476	0.0104	0.0378	0.0413
		(0.168)	(0.183)	(0.0338)	(0.0330)	(0.0336)
Prod_manu		0.223***	0.212***	0.243***	0.221***	0.207***
		(0.0405)	(0.0405)	(0.0144)	(0.0146)	(0.0156)
Prod_ser		-0.0259	0.0410	-0.0588	0.0267	0.0589
		(0.207)	(0.211)	(0.0406)	(0.0424)	(0.0464)
Constant	44.19***	49.81***	34.76***	53.73***	36.13***	30.38***
	(5.069)	(3.778)	(7.417)	(1.915)	(3.787)	(4.168)
Observations	579	579	579	542	542	523
Number of con	17	17	17	17	17	17
FE	Yes	Yes	Yes	Yes	Yes	Yes

Robust standard errors in parentheses
*** p < 0.01, ** p < 0.05, * p < 0.1

coefficient of the interaction term is significantly negative, which means the negative effect of income inequality is getting stronger as the income level increases.

In order to control the reverse causality issue[9] I also run the regressions with instrumental variables (IV), where the lagged values of $Inequality_{it}$ and $GDP_{it}*Inequality_{it}$ are treated as instruments for the current values of these two variables. I use first-order lagged values of $Inequality_{it}$ and the interaction term as instrumental variables in column (4) and (5), and second-order lagged values of these two variables in column (6). The basic results are quite robust.

2.3. Summary

The empirical evidences suggest that inequality has a negative effect on services share, and the negative effect increases in income levels. This fact is not well explained in the literature of structural change. The existing papers always assume there is a representative agent to abstract the fact of income distribution. However, with non-homothetic preferences, there is not necessarily a representative agent in the economy. Therefore, the literature assume the utility functions follow the Gorman form, so that a representative agent always exists, as in Kongsamut et al. (2001) and Herrendorf et al. (2013b). In other words, the preferences imply that the demand on a particular good is a linear function of total expenditure. As a result, although the preferences are non-homothetic in the sense that the demand curves do not pass through the origin, income distribution does not play any role in the determination of aggregate demand. Even the models with very general preferences, say, the Almost Ideal Demand System (AIDS) developed by Deaton and Muellbauer (1980), cannot explain the data patterns in the empirics. In these models, the demand is a convex function of total expenditure, this means income inequality would increase the total demand for the goods with income elasticity larger than 1 or luxury goods, given that the average income level is fixed. As a result, these preferences should always predict significantly positive relationships between income inequality and services shares. Therefore, a new model consistent with the data is needed.

3. Model

This section will first set up a static general equilibrium model, and then solve the equilibrium. I will show the income elasticities at the aggregate level is very different from its behavior at the individual level. Within this framework, I analyze the impact of income inequality on the total consumption of a particular good. The basic setup follows Foellmi and Zweimuller (2008).

[9] It is possible that the rising of service sector induces income inequality. The development of services sector may cause wage differentials between skilled and unskilled workers, as well as the polarization of the labor market, as in Buera and Kaboski (2012) and Autor and Dorn (2013) for the case of the U.S.

3.1. Setup of the model

3.1.1. Technology

Since the focus of this paper is on the demand side, I will simplify the environment of the production side. There are infinite number of industries producing goods and services in the economy, and each industry produces one good. Each product is produced using the same constant-return-to-scale technology in a competitive market. Labor is the only input in the production function of product i, $F_i(A(i), L(i))$, where $A(i)$ and $L(i)$ are the productivity level and employment level in industry i, respectively. For simplicity, $A(i)$ are assumed to be the same across industries, denoted by A. Since the production technologies are all the same, it implies that the prices of all the goods are equal, which I normalize to 1. That is, $p(i) = p \equiv 1$, for each i.

Labor is fully mobile across sectors. The wage level, w, is given by the marginal revenue of outputs with respect to labor inputs,

$$w = F_{i'}(A, L(i)).$$

3.1.2. Preference

The consumption of goods i by individual k is denoted by $c_k(i)$. The utility of individual k, u_k, depends on his consumption bundle $\{c_k(i)\}$, where i is an index of how the goods are ranked. Goods with low-i are those needed to be satiated earlier, and have high priority.[10] Everybody will rank the products in the same way. The key feature of the utility function is that the marginal utility of zero consumption of a product is finite.[11] Therefore, consumers might not buy all the products, and this generates the extensive margin. As income changes, consumers not only adjust the demand for each product, but also the number of products. In contrast, in CES utility function, the marginal utility of zero consumption is infinite, which implies that consumers always buy all the products. Similarly, Kongsamut et al. (2001) and Herrendorf et al. (2013b) and the AIDS model all implicitly assume that every products are consumed by each individual. So there are no extensive margins in these papers. As it will turn out, the extensive margin is important to deliver the theoretical predictions that are consistent with data.

In this paper, the utility function takes the form as follows,

$$u_k(\{c_k(i)\}) = \int_0^\infty i^{-\gamma}\frac{1}{2}[s^2 - (s - c_k(i))^2]di \tag{1}$$

$$s.\ t.\ \int_0^\infty p(i)c_k(i)di = e_k, \text{ and } c_k(i) \geq 0.$$

The term $\frac{1}{2}[s^2 - (s - c_k(i))^2]$ is the "baseline" utility of good i for consumer k, and the term $i^{-\gamma}$ is the weight of the baseline utility of good i, where $\gamma \in (0, 1)$.[12] The baseline utilities are the same for each good. The utility function is hierarchical in the sense that different goods receive different weights. Most essential goods for everyday life get higher weights. Therefore, goods with low i will be consumed first. s is a parameter which captures the satiation level. As it will turn out later, this utility function implies a non-linear Engel curve, which is empirically reasonable, as suggested by Lewbel (2006). The total expenditure for individual k is e_k.

Each individual will provide l_k units of labor in elastically every period. The level of l_k is exogenous given for each individual, drawn from a distribution with $c.\ d.\ f.\ G(l)$. The total labor supply in the economy is $L = \int l_k dG(l)$. One could consider l_k as the effective units of labor, $e.\ g.$ labor embodied with human capital. There is only labor income for each individual. Since this is a static model, there is no saving. The total expenditure is equal to total income.

The first order conditions with respect to $c_k(i)$ are

$$c_k(i) = s - i^\gamma\lambda_k p(i) = s - i^\gamma\lambda_k, \tag{2}$$

where λ_k is the Lagrangian multiplier for individual k, or the marginal utility of income.

The labor will be fully employed, satisfying

$$\int_0^\infty L(i)di = L, \tag{3}$$

Recall that the production technology are all the same. So the allocation of $L(i)$ is directly depending on the total demand for each product.

3.1.3. Equilibrium

In the static competitive equilibrium,

- the consumption bundle $\{c_k(i)\}$ will maximize the utility of individual k,
- all the firms will maximize their profits, and the profits will be zero,
- the labor and goods markets are clear.

For the demand side, through Eq. (2), one can easily get that the demand for a good is decreasing as i increases. So, there exists a good N, such that for $i \geq N$, the constraint $c_k(i) \geq 0$ will be binding. Suppose n_k is the minimum value of such a N for individual k. Therefore, $c_k(n_k) = 0$, which will help to pin down how many kinds of goods individual k will consume. Through Eq. (2), we can get

$$s - n_k^\gamma\lambda_k = 0.$$

Solve λ_k, and plug it into Eq. (2),

$$\begin{aligned} c_k(i) &= s\left[1 - \left(\frac{i}{n_k}\right)^\gamma\right], \text{ for} i \leq n_k, \\ c_k(i) &= 0, \qquad\qquad \text{for} i \geq n_k. \end{aligned} \tag{4}$$

That is, only consumers with income high enough will buy product i,. Therefore, the demand changes in both intensive and extensive margins with a change in income.

Now, I put the demand of each good into the budget constraint to get n_k,[13]

$$n_k = \frac{\gamma + 1}{s\gamma}e_k = e_k B \tag{5}$$

where $B = \frac{\gamma+1}{s\gamma}$..

Therefore, given e_k, we can immediately get the range of goods individual k consumes. The consumption of each good now becomes

$$\begin{aligned} c_k(i) &= s\left[1 - \left(\frac{i}{e_k B}\right)^\gamma\right], \text{ for} i \leq n_k, \\ c_k(i) &= 0, \ \text{ for } i \geq n_k. \end{aligned} \tag{6}$$

The demand function is a concave function in income, as shown in Fig. 2.

Proposition 1. *As income, e_k, increases, the income elasticity of good i for individual k will decrease.*

$$\frac{\partial c_k(i)}{\partial e_k}\frac{e_k}{c_k(i)} = \frac{\gamma}{\left(\frac{B}{i}\right)^\gamma e_k^\gamma - 1} = \frac{\gamma}{\left(\frac{n_k}{i}\right)^\gamma - 1} \tag{7}$$

Through this proposition, it's clear that this utility function implies a non-linear Engel curve. When individual k just begins to consume good i, the income elasticity is close to infinite. As income increases, the

[10] As in Foellmi and Zweimuller (2008), we could think of low-i items as agricultural goods, medium-i items as manufacturing goods, and high $-i$ items as services.

[11] Actually, any utility function satisfying this property could be used in the analysis below. The reason to choose the one used in Foellmi and Zweimuller (2008) is just for simplicity.

[12] The assumption of $\gamma \in (0, 1)$ is to ensure the integral term is finite.

[13] See Appendix A.1 for the proof.

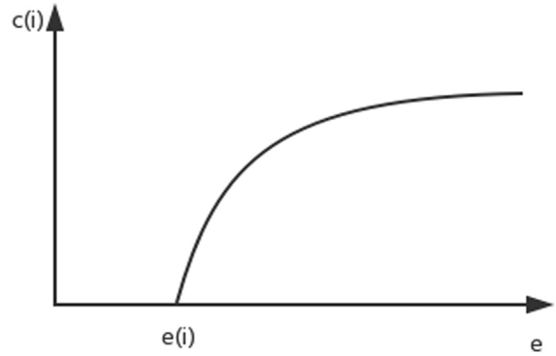

Fig. 2. The individual demand function for product *i*.

income elasticity will decline, eventually to below one, close to zero.

3.2. Aggregation

Since there is only labor income, and the wage of effective labor is equalized across sectors, the income distribution is the same as the labor endowment distribution, that is, $G(e_k)$, on the support $[e_{low,}\infty)$.

For the particular good i, the aggregate demand $X(i)$ is

$$X(i) = \int_{e_{low}}^{\infty} c_k(i) dG(e_k).$$

From Eq. (6), we know that it is possible that not every individual can afford the particular good i. Only individuals with $n_k \geq i$, or $e_k \geq e_i = \frac{i}{B}$, will consume good i, where e_i is the income level with which the individuals just begin to consume good i. The demand for good i is zero for all the individuals with income lower than e_i. Therefore, if $e_i \geq e_{low}$,

$$X(i) = \int_{e_{low}}^{e_i} 0 dG(e_k) + \int_{e_i}^{\infty} s\left[1 - \left(\frac{i}{e_k B}\right)^{\gamma}\right] dG(e_k) \tag{8}$$

$$= s(1 - G(e_i)) - s\left(\frac{i}{B}\right)^{\gamma} \int_{e_i}^{\infty} \frac{1}{e_k^{\gamma}} dG(e_k);$$

if $e_i \leq e_{low}$, everyone will consume good i,

$$X(i) = \int_{e_{low}}^{\infty} s\left[1 - \left(\frac{i}{e_k B}\right)^{\gamma}\right] dG(e_k). \tag{9}$$

In order to get closed-form solutions, I assume that income follows Pareto distribution. Actually, the Lorenz Curve underlying the Gini coefficient could be characterized by Pareto distribution, and the so-called 20–80 principle[14] is also a special case of Pareto distribution. The $C.D.F.$ and $p.d.f.$ of the distribution on the support of $[e_{low,}\infty)$[15] are

$$G(e_k) = 1 - \left(\frac{e_{low}}{e_k}\right)^{\alpha}, (\alpha > 1)$$

and

$$\frac{\alpha e_{low}^{\alpha}}{e_k^{\alpha+1}},$$

respectively, where α is the parameter governing the curvature of the distribution curve. In the context of this paper, higher α implies more equal distribution. The Mean of the distribution is $E = \frac{\alpha e_{low}}{\alpha - 1}$.

Now, Eq. (8) becomes[16]

$$X(i) = \frac{s\gamma}{\gamma + \alpha}\left(\frac{e_{low}}{e_i}\right)^{\alpha}$$
$$= \frac{s\gamma}{\gamma + \alpha}\left[\frac{E}{e_i}\frac{\alpha - 1}{\alpha}\right]^{\alpha}, \text{for} e_i \geq e_{low}. \tag{10}$$

Eq. (9) would be[17]

$$X(i) = s\left[1 - \frac{\alpha}{\gamma + \alpha}\left(\frac{e_i}{E}\frac{\alpha}{\alpha - 1}\right)^{\gamma}\right], \text{for} e_i \leq e_{low}. \tag{11}$$

Proposition 2. *The income elasticity of aggregate demand for good i is α (which is larger than 1), if good i is not consumed by everybody in the economy. Only when good i is consumed by every individual, the income elasticity of aggregate demand for good i would decrease as average income increases.*

proof. When $e_i \geq e_{low}$, the income elasticity of aggregate demand for good i is

$$\frac{\partial X(i)}{\partial E}\frac{E}{X(i)} = \frac{s\gamma}{\gamma + \alpha}\left(\frac{\alpha - 1}{\alpha}\right)^{\alpha}\alpha\left(\frac{E}{e_i}\right)^{\alpha}\frac{1}{E}\frac{E}{X(i)} = \alpha. \tag{12}$$

When $e_i \leq e_{low}$, the income elasticity of aggregate demand for good i is

$$\frac{\partial X(i)}{\partial E}\frac{E}{X(i)} = \frac{s\alpha}{\gamma + \alpha}\left(\frac{\alpha}{\alpha - 1}\right)^{\gamma}\gamma\left(\frac{e_i}{E}\right)^{\gamma}\frac{1}{E}\frac{E}{X(i)}$$
$$= \frac{\gamma}{\frac{\gamma + \alpha}{\alpha}\left(\frac{\alpha - 1}{\alpha}\frac{E}{e_i}\right)^{\gamma} - 1}. \tag{13}$$

From Proposition 2, we can see that the income elasticities of good i at aggregate level and individual level differ a lot. More specifically, when the good is not consumed by everybody, an increase in average income will not change the income elasticity. The reason is that although the income elasticity of good i decreases for existing consumers, an increase in average income introduces many new consumers. Since the new consumers have higher income elasticities, the overall effect of higher income level depends on the force whichever dominates. With Pareto distribution, the high average income elasticities of the new consumers just offset the decreasing income elasticities of the old consumers.

When $e_i < e_{low}$, the income elasticity of aggregate demand is given by Eq. (13). It is straightforward to see that as E increases, the value of elasticity decreases. Intuitively, since good i is consumed by everybody, and the income elasticity of demand at individual level is decreasing in income, the income elasticity of aggregate demand should also decrease in income levels.

3.3. The impact of income distribution on aggregate consumption

Now, I come to the main question of this paper, and analyze how income distribution affects the aggregate demand of a particular good.

I consider the case how the aggregate consumption level changes when income distribution becomes more equal across individuals, holding the average income level unchanged. In the content of Pareto distribution, more equal distribution means a higher α. If $e_i \geq e_{low}$,[18]

$$\frac{\partial X(i)}{\partial \alpha} = \frac{s\alpha}{\gamma + \alpha}\left[\frac{E}{e_i}\frac{\alpha - 1}{\alpha}\right]^{\alpha}\left[\ln\left(\frac{E}{e_i}\frac{\alpha - 1}{\alpha}\right) + \frac{1}{\alpha - 1} - \frac{1}{\gamma + \alpha}\right]; \tag{14}$$

if $e_i \leq e_{low}$,

$$\frac{\partial X(i)}{\partial \alpha} = \frac{s\gamma}{\gamma + \alpha}\frac{\alpha}{\alpha - 1}\left[\frac{\alpha}{\alpha - 1} - \frac{1}{\gamma + \alpha}\left(\frac{\alpha}{\alpha - 1}\right)^{\gamma - 1}\right] > 0. \tag{15}$$

[14] The richest 20 percents of population hold 80 percents of total wealth.
[15] We can also assume there is an upper bound for income. The results are basically the same, but it will make the calculations more tedious.
[16] See Appendix A.2 for the proof.
[17] See Appendix A.3 for the proof.
[18] See Appendix A.4.

Proposition 3. *Given average income level fixed, if a product is consumed by all individuals, more equal distribution will always induce more consumption of that good; if a product is consumed by only a fraction of people, then there is a cut-off i^*, such that if $i < i^*$, more equal distribution will increase its aggregate demand. On the other hand, for product $i > i^*$, more equal distribution will decrease its total demand, and*

$$i^* = B\frac{\alpha - 1}{\alpha}e^{\frac{1}{\alpha-1}-\frac{1}{\gamma+\alpha}}E. \tag{16}$$

Proof. For $e_i \le e_{low}$, $\frac{\partial X(i)}{\partial \alpha} > 0$ for sure.

For $e_i \ge e_{low}$, from Eq. (14), $\frac{\partial X(i)}{\partial \alpha} > 0$ requires

$$\left[\ln\left(\frac{E}{e_i}\frac{\alpha - 1}{\alpha}\right) + \frac{1}{\alpha - 1} - \frac{1}{\gamma + \alpha}\right] > 0.$$

That is,

$$\ln\left(\frac{E}{e_i}\frac{\alpha - 1}{\alpha}\right) > \frac{1}{\gamma + \alpha} - \frac{1}{\alpha - 1},$$

which implies

$$e_i < \frac{\alpha - 1}{\alpha}e^{\frac{1}{\alpha-1}-\frac{1}{\gamma+\alpha}}E.$$

Since $e_i = \frac{i}{B}$, we can get

$$i < B\frac{\alpha - 1}{\alpha}e^{\frac{1}{\alpha-1}-\frac{1}{\gamma+\alpha}}E.$$

Define

$$i^* = B\frac{\alpha - 1}{\alpha}e^{\frac{1}{\alpha-1}-\frac{1}{\gamma+\alpha}}E. \tag{17}$$

We can get Proposition 3.

Combining Proposition 2 and 3, it is clear that even the income elasticity of a product is bigger than one, it is possible that more equal distribution will induce higher total demand. It is in contrast to standard models. Even in the models with very general preferences, like AIDS, income inequality would always increase the total demand for the goods with income elasticity larger than 1 or luxury goods, given average income is fixed. In those models, to let the aggregated income elasticity bigger than one, the income elasticity of a good is larger than one for every individual and the product is consumed by all the individuals. Then, the total demand is a convex function of total expenditure. Under this circumstance, inequality is in favor of higher total demand, because the increasing demand by the richer would outweigh the decreasing demand by the poorer.

While in the current model, there is a new margin to adjust, the extensive margin. Even though the income elasticity is larger than 1for a product at the aggregate level, more inequality causes that some people cannot afford that product any more, which will greatly hurt the total demand. In other words, the decrease of demand for these poorer consumers are higher than the increase in demand for the consumers who become richer.

To understand the mechanism of the model intuitively, consider Figs. 2 and 3. Fig. 2 plots the individual demand function for product i, as Eq. (6) indicates. The minimum income level required to afford product i is $e(i)$. Consider two persons in the economy. Suppose their incomes, e, are the same, and $e > e(i)$. If there is a redistribution, their incomes become, e' and e'', respectively. If $e'' > e' > e(i)$, then inequality implies that the aggregate demand of i is lower, as the demand function is concave in income. However, if $e'' > e(i) > e'$, it is possible that the aggregate demand of the two persons is higher. Therefore, the impact of inequality on the total demand of a product could be positive or negative. Specifically, according to Proposition 3, the impact of inequality on the total demand of product i depends on i^*, as Fig. 3 displays.

Caron et al. (2014) estimated the income elasticity for about 50

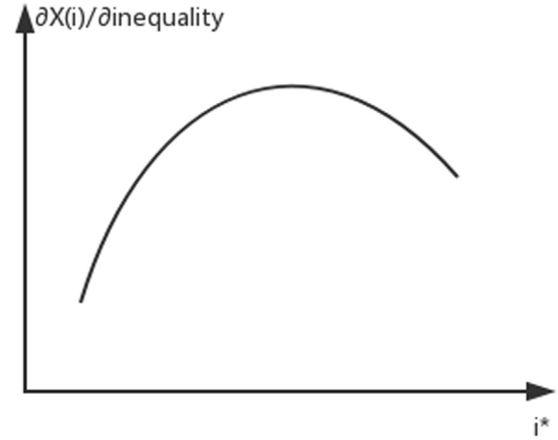

Fig. 3. The impact of inequality on the total demand for product i.

industries across agriculture, manufacturing, and services. If we rank the industries according to their income elasticity, most agriculture goods are within the range of low income elasticity, corresponding to low value of i in the model of this paper. On the other hand, most industries in services are within the range of high income elasticity, corresponding to high value of i. For manufacturing, some have small values of i, such as textiles and electricity, and some have large values of i, such as Motor vehicles, electronic equipment, and plant-based fibers.

Back to the model in this paper, if i^* is very small, then for most industries in services, i >i^*. Therefore, the total demand for services increase with inequality, according to Proposition 3. However, if i^* is large, such that for many industries in services, i <i^*, the total demand for services would decrease with inequality. Therefore, the model provides a possible explanation for the negative correlation between the employment share of services and inequality, although the income elasticity of services sector as a whole is believed to be larger than 1.

Then, I check how the impact of inequality changes as income increases.

Proposition 4. *As average income increases, the cut-off value of i^* will increase. That is, as an economy gets richer, more equal distribution will have positive effects on more products.*

Proof. From Eq. (16), it is obvious that the differentiation of i^* with respect to E is always positive.

$$\frac{\partial i^*}{\partial E} = B\frac{\alpha - 1}{\alpha}e^{\frac{1}{\alpha-1}-\frac{1}{\gamma+\alpha}} > 0.$$

Proposition 4 means that income distribution has negative impacts on more industries as average income increases. It provides an explanation for the fact that the interaction of GDP per capital and inequality has a negative impact on services share in Section Section 2. When GDP per capita is low, i^* is very small and many industries in services are in the range bigger than i^*. It indicates that the total demand for services increase with inequality. However, as income level increases, i^* becomes larger, such that more industries of the service sector lie in the range $i < i^*$. It implies that the total demand for services would gradually decrease with inequality, with income level increases. It is consistent with the regression results in column (3), (5), and (6) of Table 2.

4. Conclusion

This paper shows that income inequality has negative effect on the size of services sector, and the negative effect gets significantly stronger as income increases. These patterns could not be well explained by existing models. Then I propose a framework with non-homothetic preferences and heterogeneous agents in terms of different income levels. The key future is that consumers could adjust along both the intensive (the demand for each product) and extensive margin (the number of products). When income follows Pareto distribution, the model could be analytically solved. I show that more unequal distribution could have negative impact on the total demand for a product with income elasticity larger than 1.

Appendix A. Appendix

A.1. The Proof of Eq. (5)

According to the budget constraint for individual k,

$$e_k = \int_0^{n_k} s\left[1-\left(\frac{i}{n_k}\right)^{\gamma}\right]di = sn_k - s\int_0^{n_k}\left(\frac{i}{n_k}\right)^{\gamma}di = sn_k - \frac{s}{\gamma+1}n_k,$$

which implies

$$n_k = \frac{\gamma+1}{s\gamma}e_k.$$

A.2. The proof of Eq. (10)

Plug the C.D.F. and p.d.f. of Pareto distribution into Eq. (8), resulting

$$\begin{aligned}
X(i) = & \; s\left(\frac{e_{low}}{e_k}\right)^{\alpha} - s\left(\frac{i}{B}\right)^{\gamma}\int_{e_i}^{\infty}\frac{1}{e_k^{\gamma}}\frac{\alpha e_{low}^{\alpha}}{e_k^{\alpha+1}}de_k \\
= & \; s\left(\frac{e_{low}}{e_k}\right)^{\alpha} - s\left(\frac{i}{B}\right)^{\gamma}\alpha e_{low}^{\alpha}\int_{e_i}^{\infty}e_k^{-(\gamma+\alpha+1)}de_k \\
= & \; s\left(\frac{e_{low}}{e_k}\right)^{\alpha} - s\left(\frac{i}{B}\right)^{\gamma}\alpha e_{low}^{\alpha}\frac{1}{\gamma+\alpha}\frac{1}{e_i^{\gamma+\alpha}} \\
= & \; \frac{s\gamma}{\gamma+\alpha}\left(\frac{e_{low}}{e_i}\right)^{\alpha} \\
= & \; \frac{s\gamma}{\gamma+\alpha}\left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha}.
\end{aligned}$$

A.3. The proof of Eq. (11)

$$\begin{aligned}
X(i) = & \; \int_{e_{low}}^{\infty} s\left[1-\left(\frac{i}{e_k B}\right)^{\gamma}\right]dG(e_k) \\
= & \; s - s\left(\frac{i}{B}\right)^{\gamma}\alpha e_{low}^{\alpha}\frac{1}{\gamma+\alpha}\frac{1}{e_{low}^{\gamma+\alpha}} \\
= & \; s\left[1-\frac{\alpha}{\gamma+\alpha}\left(\frac{e_i}{e_{low}}\right)^{\gamma}\right] \\
= & \; s\left[1-\frac{\alpha}{\gamma+\alpha}\left(\frac{e_i}{E}\frac{\alpha}{\alpha-1}\right)^{\gamma}\right].
\end{aligned}$$

A.4. The proof of Eq. (14)

First, let's deal with the differentiation of the term $\left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha}$ with respect to α,

$$\begin{aligned}
\frac{\partial\left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha}}{\partial\alpha} = & \; \frac{\partial e^{\alpha\ln\left(\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right)}}{\partial\alpha} \\
= & \; e^{\alpha\ln\left(\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right)}\left[\ln\left(\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right)+\frac{1}{\alpha-1}\right] \\
= & \; \left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha}\left[\ln\left(\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right)+\frac{1}{\alpha-1}\right].
\end{aligned}$$

Then,

$$
\begin{aligned}
\frac{\partial X(i)}{\partial \alpha} = & -\frac{s\gamma}{(\gamma+\alpha)^2}\left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha} \\
& + \frac{s\gamma}{\gamma+\alpha}\left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha}\left[\ln\left(\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right)+\frac{1}{\alpha-1}\right] \\
= & \frac{s\alpha}{\gamma+\alpha}\left[\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right]^{\alpha}\left[\ln\left(\frac{E}{e_i}\frac{\alpha-1}{\alpha}\right)+\frac{1}{\alpha-1}-\frac{1}{\gamma+\alpha}\right].
\end{aligned}
$$

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